



# Drift analysis of ant colony optimization of stochastic linear pseudo-boolean functions



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## ABSTRACT

In this paper we study the behavior of a variant of the Max–Min Ant System algorithm when applied to a stochastic Linear Pseudo-Boolean Optimization problem. Previous related work is on a partial analysis of its performance on a different problem. Here, we carry out its complete performance analysis giving a bound on its average runtime using drift analysis. For the purpose, we give a new drift theorem and use it to analyze the algorithm when applied to our problem.

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## 1. Introduction

Drift analysis is an increasingly popular technique for the runtime analysis of randomized search algorithms [25,5,6]. Several variants based on variable drift theorems have already been proposed, for instance in [22,18,3]. Here, we set out to use the drift analysis method to analyze the behavior of an Ant Colony Optimization (ACO) algorithm when applied to the Linear Pseudo-Boolean Optimization (LPBO) problem. The choice of this algorithm is due to it being a powerful bio-inspired meta-heuristic. It was first described in [7]. ACO is inspired by the complex behavior of ant colonies which exhibit the so called swarm intelligence. This emergent intelligence results from the simple behavior of individual ants which, using pheromone as an indirect communication mechanism, confer to the colony a complex behavior on par with that of higher level organisms. It has been applied successfully to a wide range of problems arising in combinatorial as well as stochastic, dynamic and continuous optimization, [8,20]. Here we consider the Single-Destination Shortest Path (SDSP) problem. When the algorithm is not directly applicable to a given problem, the latter is transformed to SDSP.

In general, ACO algorithms are used when a solution is made up of many components; artificial ants then build solutions by successively selecting appropriate components. Pheromone is added to components that often belong to good solutions, while it evaporates from others. The amount of the pheromone update is controlled by the so-called evaporation factor  $\rho$ ; when set low, it leads to a slower but wider coverage of the search space. This usually allows for better solutions to be found but at the cost of a longer runtime.

There are several papers which analyze the behavior of ACO algorithms. The first study is on proofs of convergence [11,12]. Recent advances concern the runtime of ACO algorithms when applied to combinatorial problems such as the Minimum Spanning Tree (MST) [23], TSP [19], SDSP [9] and Pseudo-Boolean functions [20].

In this paper, we are particularly interested in the work of [9] which is concerned with analyzing the performance of a new version of MMAS called MMAS-fp-norm, when applied to SDSP. According to [9], MMAS-fp-norm compares well with other variants of MMAS from an experimental point of view. But, their theoretical study focuses mainly on getting an upper bound on the expected first hitting time. This is not sufficient; a definite conclusion on the performance of MMAS-fp-norm is therefore still lacking due to the missing lower bound which allows to conclude on the bad performance of the algorithm contrary to the upper bound which only allows to conclude on its good performance. Here, we endeavor to fill this gap focusing on the study of the MMAS-fp-norm algorithm

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